

# Near Tribimaximal Neutrino Mixing with $\Delta(27)$ Symmetry

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## Abstract

The discrete subgroup  $\Delta(27)$  of  $SU(3)$  has the interesting multiplication rule  $3 \times 3 = \bar{3} + \bar{3} + \bar{3}$ , which is used to obtain near tribimaximal neutrino mixing. Using present neutrino oscillation data as input, this model predicts that the effective mass  $m_{ee}$  measured in neutrinoless double beta decay will be 0.14 eV.

The non-Abelian discrete subgroup  $\Delta(12)$  of  $SU(3)$  [1], more familiarly known as  $A_4$  [2, 3], has been shown to be useful [4, 5, 6, 7, 8] for obtaining the tribimaximal mixing [9] of neutrinos, in good agreement [10] with present data. In the basis where the charged-lepton mass matrix  $\mathcal{M}_l$  is diagonal, the Majorana neutrino mass matrix is given by [4]

$$\mathcal{M}_\nu = \begin{pmatrix} x + y + z & -x & -x \\ -x & y & z \\ -x & z & y \end{pmatrix} \quad (1)$$

which is form diagonal, i.e. it is diagonalized by

$$U_3 = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}, \quad (2)$$

independent of its three mass eigenvalues

$$m_1 = 2x + y + z, \quad m_2 = -x + y + z, \quad m_3 = y - z. \quad (3)$$

Since the values of  $x, y, z$  are arbitrary, neutrino masses are not predicted in such a scheme. [However, they can be restricted in two special cases: (A)  $y = 2x$  [5], and (B)  $(x + y)^2 = (2x - y)(x + z)$  [6].]

In this paper, using the discrete group  $\Delta(27)$  [11, 12, 13] which is next in the sequence of  $\Delta(3n^2)$  subgroups of  $SU(3)$ , it will be shown that  $\mathcal{M}_\nu$  deviates slightly from Eq. (1) such that the change of  $\tan^2 \theta_{12}$  from 0.5 to 0.45 allows the prediction of  $m_{ee} = 0.14$  eV for the effective neutrino mass in neutrinoless double beta decay, a value accessible in the next generation of such experiments [14].

The non-Abelian discrete group  $\Delta(27)$  has 27 elements divided into 11 equivalence classes. It has 9 one-dimensional irreducible representations  $\mathbf{1}_i (i = 1, \dots, 9)$  and 2 three-dimensional ones  $\mathbf{3}$  and  $\bar{\mathbf{3}}$ . Its character table and the 27 defining  $3 \times 3$  matrices are given in Ref. [12]. Its group multiplication rules are

$$\mathbf{3} \times \mathbf{3} = \bar{\mathbf{3}} + \bar{\mathbf{3}} + \bar{\mathbf{3}}, \quad \text{and} \quad \mathbf{3} \times \bar{\mathbf{3}} = \sum_{i=1}^9 \mathbf{1}_i, \quad (4)$$

where

$$\mathbf{1}_1 = 1\bar{1} + 2\bar{2} + 3\bar{3}, \quad \mathbf{1}_2 = 1\bar{1} + \omega 2\bar{2} + \omega^2 3\bar{3}, \quad \mathbf{1}_3 = 1\bar{1} + \omega^2 2\bar{2} + \omega 3\bar{3}, \quad (5)$$

$$\mathbf{1}_4 = 1\bar{2} + 2\bar{3} + 3\bar{1}, \quad \mathbf{1}_5 = 1\bar{2} + \omega 2\bar{3} + \omega^2 3\bar{1}, \quad \mathbf{1}_6 = 1\bar{2} + \omega^2 2\bar{3} + \omega 3\bar{1}, \quad (6)$$

$$\mathbf{1}_7 = 2\bar{1} + 3\bar{2} + 1\bar{3}, \quad \mathbf{1}_8 = 2\bar{1} + \omega^2 3\bar{2} + \omega 1\bar{3}, \quad \mathbf{1}_9 = 2\bar{1} + \omega 3\bar{2} + \omega^2 1\bar{3}, \quad (7)$$

with  $\omega = \exp(2\pi i/3)$ , i.e.  $1 + \omega + \omega^2 = 0$ .

Let the lepton doublets  $(\nu_i, l_i)$  as well as singlets  $l_i^c$  transform as  $\mathbf{3}$  under  $\Delta(27)$ , then with three Higgs doublets  $(\phi_i^0, \phi_i^-)$  also transforming as  $\mathbf{3}$ , the charged-lepton mass matrix is of the form

$$\mathcal{M}_l = \begin{pmatrix} h_1 v_1 & h_2 v_3 & h_3 v_2 \\ h_3 v_3 & h_1 v_2 & h_2 v_1 \\ h_2 v_2 & h_3 v_1 & h_1 v_3 \end{pmatrix}. \quad (8)$$

As shown in Ref. [15], if  $v_1 = v_2 = v_3$ , this is also form diagonal, i.e.

$$\mathcal{M}_l = U_L \begin{pmatrix} (h_1 + h_2 + h_3)v & 0 & 0 \\ 0 & (h_1 + h_2\omega + h_3\omega^2)v & 0 \\ 0 & 0 & (h_1 + h_2\omega^2 + h_3\omega)v \end{pmatrix} U_R^\dagger, \quad (9)$$

where  $U_L = U_R$  is the familiar

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad (10)$$

first introduced by Cabibbo [16] and Wolfenstein [17].

At the same time, with three Higgs triplets  $(\xi^{++}, \xi^+, \xi^0)$  transforming as  $\mathbf{3}$ , the Majorana neutrino mass matrix is of the same form as Eq. (8) but it has to be symmetric, i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} f_1 u_1 & f_2 u_3 & f_2 u_2 \\ f_2 u_3 & f_1 u_2 & f_2 u_1 \\ f_2 u_2 & f_2 u_1 & f_1 u_3 \end{pmatrix}. \quad (11)$$

In this basis, the condition for tribimaximal mixing is  $u_2 = u_3 = 0$ , but then the mass eigenvalues become  $m_1 = f_1 u_1$ ,  $m_2 = f_2 u_1$ , and  $m_3 = -f_2 u_1$ , which are of course not

realistic. On the other hand, this represents a symmetry limit, and small deviations from it will allow the masses to be different, correlated with changes in the mixing angles from those of tribimaximal mixing. In the following, it is shown how present data will predict  $m_{ee} = 0.14$  eV in the context of this model.

Given the form of Eq. (11), let it be rewritten as

$$\mathcal{M}_\nu = \begin{pmatrix} \lambda d & f & e \\ f & \lambda e & d \\ e & d & \lambda f \end{pmatrix} = U_2 \begin{pmatrix} d + \lambda(e+f)/2 & (e+f)/\sqrt{2} & \lambda(-e+f)/2 \\ (e+f)/\sqrt{2} & \lambda d & (e-f)/\sqrt{2} \\ \lambda(-e+f)/2 & (e-f)/\sqrt{2} & d - \lambda(e+f)/2 \end{pmatrix} U_2^T, \quad (12)$$

where

$$U_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & i \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -i/\sqrt{2} \\ 1/\sqrt{2} & 0 & i/\sqrt{2} \end{pmatrix}, \quad (13)$$

then  $U_3$  of Eq. (2) is obtained from  $U_\omega$  of Eq. (10) and the above, i.e.  $U_3 = U_\omega^\dagger U_2$ . This means that tribimaximal mixing is approximately obtained provided that  $e, f \ll d$ .

To obtain  $\Delta m_{sol}^2 \ll \Delta m_{atm}^2$ , set

$$d + \frac{\lambda}{2}(e+f) = -\lambda d + \delta, \quad (14)$$

where  $\delta > 0$  is small and  $\lambda d > 0$  has been assumed. Then

$$m_{1,2} = \frac{\delta}{2} \mp m_0, \quad (15)$$

where  $m_0 > 0$  with

$$m_0^2 = \left( \lambda d - \frac{\delta}{2} \right)^2 + \frac{2}{\lambda^2} [(1+\lambda)d - \delta]^2. \quad (16)$$

Hence

$$m_2^2 - m_1^2 = 2\delta m_0 > 0, \quad (17)$$

and

$$m_3^2 - m_0^2 = \frac{2d^2}{\lambda^2} (\lambda^2 - 1)(2\lambda + 1). \quad (18)$$

The new  $\theta_{12}$  is now given by

$$\tan \theta_{12} \simeq \frac{1}{\sqrt{2}} \left[ 1 + \frac{3}{2} \left( \frac{1+\lambda}{\lambda^2} \right) \right] \simeq \frac{1}{\sqrt{2}} \left[ 1 + \frac{3\epsilon}{2} \right], \quad (19)$$

where  $\lambda = -1 + \epsilon$  has been used. Using the experimental central value of 0.45 for  $\tan^2 \theta_{12}$ ,

$$\epsilon \simeq -0.034 \quad (20)$$

is obtained. Since  $m_3^2 - m_0^2 \simeq 4\epsilon d^2$ , this means that an inverted ordering of neutrino masses is predicted. Furthermore, since  $m_{ee} \simeq |d|$ , the experimental central value of  $2.7 \times 10^{-3}$  eV for  $|\Delta m_{atm}^2|$  implies

$$m_{ee} = \left| \frac{\Delta m_{atm}^2}{4\epsilon} \right|^{1/2} = 0.14 \text{ eV}. \quad (21)$$

If  $\tan^2 \theta_{12} = 0.4$  is used instead, then  $m_{ee} = 0.1$  eV.

As for  $\theta_{13}$ , it is given here by

$$\sin \theta_{13} \simeq (\sqrt{2} \sin \theta_{12} - \cos \theta_{12}) \left( \frac{e-f}{2d} \right) \simeq -0.02 \left( \frac{e-f}{d} \right). \quad (22)$$

Since only  $(e+f)/d \simeq 2\epsilon$  has been determined, there is no prediction for  $\theta_{13}$  in this model.

In conclusion, the family symmetry  $\Delta(27)$  has been discussed in a simple model as the origin of the observed mixing pattern of neutrinos. It is able to describe present data and has a specific prediction of the effective neutrino mass, i.e. 0.14 eV, in neutrinoless double beta decay.

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